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Kähler moduli double inflation

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Abstract

We show that double inflation is naturally realized in Kähler moduli inflation, which is caused by moduli associated with string compactification. We find that there is a small coupling between the two inflatons which leads to amplification of perturbations through parametric resonance in the intermediate stage of double inflation. This results in the appearance of a peak in the power spectrum of the primordial curvature perturbation. We numerically calculate the power spectrum and show that the power spectrum can have a peak on observationally interesing scales. We also compute the TT-spectrum of CMB based on the power spectrum with a peak and see that it better fits WMAP 7-years data.

1 Introduction

The inflation is now a standard paradigm of the early universe. It not only solves many problems associated with the early universe but also explains the origin of the density perturbation of the universe. That is, the inflation generates the density perturbation from the quantum fluctuation of inflaton, the scalar field whose potential energy drives exponential expansion of the universe. Search for the inflationary model which is based on particle physics has become one of the main topics of cosmology. In particular, recently, inflation models realized in framework of superstring theory are studied vigorously. Superstring theory predicts the existence of many scalar fields, moduli, in the low-energy effective theory and some of them can be candidates of inflaton(See [1, 2, 3, 4, 5] for comprehensive reviews) .

Vacuum expectation values (VEVs) of some of moduli determine the geometry of the extra dimensions. It is believed that in the vacuum all moduli are stabilized at the minimum of their potential and the extra dimensions are compactified to a specific Calabi-Yau manifold. Comprehension of how to stabilize moduli, especially in the context of type-IIB superstring, has greatly advanced in this decade [6, 7] (see also [8] for a review and references therein). Following the stabilization mechanism, we can derive the potential of moduli. It was found that some of moduli may be apart from their minimum at first and slowly roll down toward the minimum along the potential in the early universe, which leads to the sufficiently long inflation [9, 10, 11, 12, 13]. Among inflation models driven by various moduli, here, we concentrate on Kähler moduli inflation, which was first considered in [11] (see elso [14]).

Each Kähler modulus, $T_i = \tau_i + i\theta_i$ is associated with a 4-cycle on the Calabi-Yau manifold. The real part τ_i represents the volume of the 4-cycle and the imaginary part θ_i is the axionic component. Kähler moduli inflation is based on the Large-volume flux compactification [15, 16], which is a different compactification mechanism from KKLT scenario [7]. In this mechanism, the total volume of the extra dimension is exponentially large, and the Kähler modulus with largest real part is responsible for the total volume. The real part of another Kähler modulus may become an inflaton.

For general Calabi-Yau manifolds, the number of the Kähler moduli is more than one. In fact, the Kähler moduli inflation requires at least three Kähler moduli. The first one corresponds to the total volume, the second one is needed to stabilize the first one and

the third one is an inflaton. However, there is no reason that only three Kähler moduli exist and we expect more than three Kähler moduli. The models in which the total number of Kähler moduli is four and two of them behave as inflatons were considered in [17, 18]. Therefore, in this paper, we consider such situation that two Kähler moduli play a role of inflatons and hence two stages of the inflation take place. Especially, we assume that the duration of the second inflation is not long enough to erase the effect of the first inflation. Such a two-stage inflationary model driven by two scalar fields is called double inflation [19, 20, 21]. In the simplest double inflation, each inflaton has no(or only weak) coupling to another, so that the dynamics of the first inflaton does not affect the movement of second one. In other words, the potential of inflatons must has a form like

$$V(\phi_1, \phi_2) = V_1(\phi_1) + V_2(\phi_2). \tag{1}$$

This form is naturally realized for Kähler moduli. This, in addition to the plethora of moduli, means that the double inflation, or the inflation which consists of more than two stages is natural rather than possible in superstring theory.

In this paper, we investigate the power spectrum of the density perturbation generated during the double inflation caused by two Kähler moduli. The perturbations produced by various models of the double inflation were studied in many papers [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29]. The power spectrum of the density perturbation from the double inflation has features which do not exist in that from the single-field inflation. The Fourier modes of the perturbation which exit Hubble horizon during the first inflation have different amplitude and tilt from those which exit horizon during the second inflation. Besides, as pointed out in [29], the power spectrum might have a characteristic peak due to the parametric resonance [30, 31, 32]. During the intermediate stage between two inflationary stages, the first inflaton oscillates around the minimum of its potential, and its fluctuation can grow exponentially through parametric resonance. Then, if the first inflaton has a small coupling to the second one, forced oscillation occurs and the fluctuation of the second inflaton also grows exponentially. After the parametric resonance becomes inefficient, the amplified fluctuation of the first inflaton decreases by the cosmic expansion. On the other hand, since the second inflaton is light during the second stage, its fluctuation remains amplified. Therefore, the curvature perturbation is also amplified and its Fourier modes which exit the horizon with large amplitude sustain it outside the horizon. Modes affected by this resonance effect are only those which exist in the

resonance band while the first inflaton oscillates with large amplitude. Then, the resultant power spectrum has a sharp peak. We perform numerical integration of the system of the equations which describe the evolution of various perturbation variables during the inflation, and compute the power spectrum generated by the double inflation by two Kähler moduli. We show that in this model the power spectrum has a peak of the type explained above.

Next, as one of implications of the double moduli inflation, we discuss the influence of this peak on the spectrum of the temperature fluctuation of the cosmic microwave background(CMB). It is known that there is an upward deviation of the observational value compared to the predicted value around $\ell \simeq 40$ [33]. We attribute this deviation to the existence of the small peak in the inflationary power spectrum produced by the above mechanism, and show that the TT spectrum fits better around $\ell \simeq 40$.

This paper is organized as follows. In Sec. 2, we review Kähler moduli inflation. We present the equations which describe the evolution of the perturbations and the formalism to compute the power spectrum in Sec. 3. In Sec. 4, we numerically integrate the system of the equation in Sec. 3 and obtain the power spectrum. Here, we see that it actually has a peak. In Sec. 5, we compute the TT spectrum of CMB perturbation using the power spectrum obtained in Sec. 4 and fit the theoretical spectrum to the observational data, especially around $\ell \simeq 40$. We summarize this paper in Sec. 6.

Throughout this paper, we use the Planck unit, i.e. $M_p = 1$.

2 Review of Kähler moduli inflation

In this section, let us briefly review Kähler moduli inflation [11, 14]. Kähler moduli inflation is based on IIB flux compactification. In this framework, as a low-energy effective theory, we can consider 4-dimensional $\mathcal{N} = 1$ supergravity characterized by the following superpotential and Kähler potential in the string frame:

$$W = W_0 + \sum_{i=2}^{h^{1,2}} A_i e^{-a_i T_i} \tag{2}$$

$$K = K_{cs} - 2\ln\left(\mathcal{V} + \frac{\xi}{2}\right). \tag{3}$$

Here, W_0 is the superpotential for complex structure moduli and dilaton, which is induced by the background flux on the Calabi-Yau manifold M, and K_{cs} is their Kähler potential.

We assume that these moduli are stabilized at higher energy scale than that of the inflation. Then we can treat W_0 and K_{cs} as constants. The second term in (2) is for Kähler moduli and arises from the non-perturbative effect. $h^{1,2}$ is the Hodge number of M and equal to the number of the Kähler moduli. A_i and a_i are model-dependent constants. (The reason why (2) does not have the contribution from T_1 is described below.) $\mathcal{V}$ is the volume of M in the unit of string length scale l_s and we assume that $\mathcal{V}$ has a specific form as

$$
\begin{aligned}
\mathcal{V} &= \alpha \left(\tau_1^{3/2} - \sum_{i=2}^{h^{1,2}} \lambda_i \tau_i^{3/2} \right) \\
&= \frac{\alpha}{2\sqrt{2}} \left[(T_1 + \bar{T}_1)^{3/2} - \sum_{i=2}^{h^{1,2}} \lambda_i (T_i + \bar{T}_i)^{3/2} \right],
\end{aligned}
\tag{4}
$$

where α and λ_i are constants which depend on M. The term $\frac{\xi}{2}$ in the logarithm arises from α'-correction [34], where $\xi = -\frac{\zeta(3)\chi(M)}{2(2\pi)^3}$ and $\chi(M)$ is the Euler-number of M. We assume that $\xi > 0$.

Given (2) and (3), we can derive the potential of the Kähler moduli using the formula

$$
V = e^K (K^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2),
\tag{5}
$$

where $D_i = \partial_i + K_i$, the subscript $i(\bar{i})$ means the derivative by $T_i(\bar{T}_i)$ and $K^{i\bar{j}}$ is the inverse of the matrix $K_{i\bar{j}}$. The resultant potential, including the prefactor which arises from the conversion from the string frame to the Einstein frame, is obtained as[1]

$$
\begin{aligned}
V = \left(\frac{g_s^4}{8\pi} \right) &\left[\sum_{\substack{i,j=2 \\ i<j}}^{h^{1,2}} \frac{A_i A_j \cos(a_i \theta_i - a_j \theta_j)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} e^{-(a_i \tau_i + a_j \tau_j)} \left(32(2\mathcal{V} + \xi)(a_i \tau_i + a_j \tau_j + 2 a_i a_j \tau_i \tau_j) + 24\xi \right) \right. \\
&+ \frac{12 W_0^2 \xi}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \sum_{i=2}^{h^{1,2}} \left\{ \frac{12 e^{-2a_i \tau_i} \xi A_i^2}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} + \frac{16(a_i A_i)^2 \sqrt{\tau_i} e^{-2a_i \tau_i}}{3\alpha \lambda_i (2\mathcal{V} + \xi)} \right. \\
&+ \left. \left. \frac{32 e^{-2a_i \tau_i} a_i A_i^2 \tau_i (1 + a_i \tau_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} + \frac{8 W_0 A_i e^{-a_i \tau_i} \cos(a_i \theta_i)}{(4\mathcal{V} - \xi)(2\mathcal{V} + \xi)} \left(\frac{3\xi}{2\mathcal{V} + \xi} + 4 a_i \tau_i \right) \right\} \right],
\end{aligned}
\tag{6}
$$

where g_s is the string coupling constant. Generally, the minimum value of this potential is negative. In order that the minimum value, i.e., the vacuum energy after the inflation,

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[1]We make the factor $e^{K_{cs}}$ absorbed in other constants such as W_0 and A_i.

becomes zero, we have to add the uplifting term, for example, the contribution from anti-D3 branes[7]. For the natural values of the parameter, $\mathcal{V} \gg 1$ and $\tau_1 \gg \tau_2, \tau_3, ...$ at the minimum of (6).

That is, the extra dimension is compactified so that its volume is extremely large. Therefore, $e^{-a_1 T_1}$ is much smaller than unity and hence the term containing this factor is omitted from (2).

$\tau_2, \tau_3, ...$ serve for stabilization of $\mathcal{V}$. However, if

$$\frac{\lambda_n / a_n^{3/2}}{\sum_{i=2}^{h^{1,2}} \lambda_i / a_i^{3/2}} \ll 1 \tag{7}$$

is satisfied for some n, τ_n can leave its minimum without destabilizing $\mathcal{V}$. When τ_n is away from the minimum, the terms containing τ_n in the potential are extremely small, because of the exponential factor $e^{-a_n \tau_n}$. This make the potential for τ_n flat enough to realize the slow roll inflation[2] . From now, we assume that the two real parts τ_2, τ_3 of Kähler moduli are inflatons and that all imaginary parts θ_i and the other real parts are stabilized at their minimum of the potential[3] . In our scenario there has to exist at least four Kähler moduli (A concrete example of a Calabi-Yau manifold which has four Kähler moduli can be found in [35]). We treat $\mathcal{V}$ as a constant since τ_1 is stabilized and $\tau_1 \gg \tau_2, \tau_3$. $e^{-a_i \tau_i} \sim \mathcal{V}^{-1}$ near the minimum, and if we take the leading terms in the expansion into power series of $\mathcal{V}^{-1}$, we obtain

$$V^{(3)} = \left(\frac{g_s^4}{8\pi}\right)\left(V_0 - \sum_{i=2,3} \frac{4 W_0 a_i A_i}{\mathcal{V}^2} \tau_i e^{-a_i \tau_i} + \sum_{i=2,3} \frac{8(a_i A_i)^2}{3\alpha \lambda_i \mathcal{V}} \sqrt{\tau_i} e^{-2a_i \tau_i}\right) \tag{8}$$

where V_0 is the part which does not depend on $\tau_i (i \geq 2)$, including the uplifting term. It is apparent that the potential for each τ_i separates from the other and the potential takes the form of (1). During the inflation, when the inflaton is away from the minimum, the

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[2]The string loop correction to the Kähler potential might spoil the flatness of the potential of Kähler moduli and the fine tuning of the parameters are required in order for the inflation to occur[36, 37, 38]. However, we neglect this subtlety and assume that the flatness is maintained.

[3] In fact, the mass of the imaginary part θ_i is comparable to that of the real part τ_i and the motion in the direction θ_i may be as important as that in the direction τ_i [12]. In most inflationary trajectories, both τ_i and θ_i evolve at first, then at some point θ_i gets trapped at the minimum of the potential and after that the field straightly moves in the direction of τ_i. In addition, this type of inflationary models can easily make the inflation much longer than 50 or 60 e-foldings by setting the initial position of the inflaton apart from the minimum of the potential. Therefore, the situation where all imaginary parts have fallen into the minimum by the last 50 or 60 e-foldings is not special and we can assume such a situation.
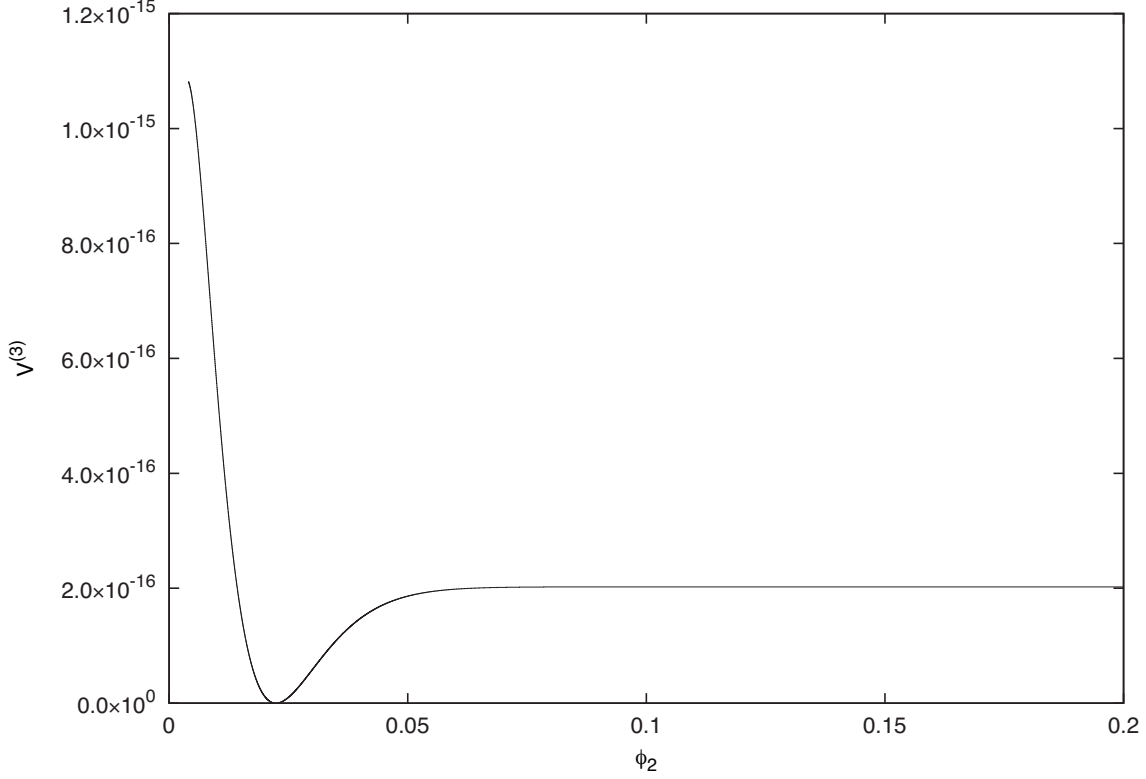
Figure 1: The potential of ϕ_2 in the leading order of the expansion into power series of $\mathcal{V}^{-1}$ under the parameter set shown in TABLE 1. The constant term is added such that the minimum value is equal to 0.

third term in (8) can be also neglected. The shape of the potential of the single Kähler moduli is depicted in Fig. 1. This potential is extremely flat for large field value, but near the minimum it is much steeper. This feature of the potential leads to the rapid oscillation of the first inflaton about its minimum in the intermediate stage of the inflation.

Although we can basically understand the dynamics of the inflaton using only the leading part $V^{(3)}$ in the potential, there is the subleading part given by

$$V^{(4)} = \left(\frac{g_s^4}{8\pi} \right) \Bigg[- \frac{4A_2 A_3}{\mathcal{V}^2} (a_2 \tau_2 + a_3 \tau_3 + 2a_2 a_3 \tau_2 \tau_3) e^{-a_2 \tau_2 - a_3 \tau_3}$$
$$+ \sum_{i=2,3} \Bigg\{ - \frac{4\xi(a_i A_i)^2 \sqrt{\tau_i}}{3\alpha\lambda_i \mathcal{V}^2} e^{-2a_i \tau_i} + \frac{4a_i A_i^2 \tau_i (1 + a_i \tau_i)}{\mathcal{V}^2} e^{-a_i \tau_i}$$
$$- \frac{3\xi W_0 A_i}{2\mathcal{V}^3} e^{-a_i \tau_i} + \frac{\xi W_0 a_i A_i \tau_i}{\mathcal{V}^3} e^{-a_i \tau_i} \Bigg\} \Bigg].$$
(9)

In this order, the coupling between τ_2 and τ_3 exists. These term do not affect the 0-th order dynamics of inflatons. However, the small coupling in this order largely affects the evolution of the fluctuation of inflatons, as we see below.

The nontrivial form of Kähler potential (3) makes the kinetic term for inflatons non-canonical. The kinetic term is $-K_{i\bar{j}}D_\mu T^i D^\mu \bar{T}^{\bar{j}}$. In the leading order of $\mathcal{V}^{-1}$, $K_{i\bar{j}}$ is given by

$$K_{i\bar{j}} \simeq \frac{3\alpha\lambda_i}{8\mathcal{V}\sqrt{\tau_i}}\delta^{ij}. \tag{10}$$

We neglect subleading contributions to $K_{i\bar{j}}$ for simplicity. For $K_{i\bar{j}}$ as (10), we obtain the canonical kinetic term by redefinition of fields as follows,

$$\phi_i = \sqrt{\frac{4\alpha\lambda_i}{3\mathcal{V}}}\tau_i^{3/4}. \tag{11}$$

3　Formalism to compute the fluctuation of inflatons

Here, we present the formalism [22] that we adopt to trace the evolution of the fluctuations of the inflatons and calculate resultant cosmological perturbations.

With use of the Newtonian gauge the perturbed space-time is described as

$$ds^2 = -(1 - 2\Phi(t, \vec{x}))dt^2 + a^2(t)(1 + 2\Phi(t, \vec{x}))d^2\vec{x}, \tag{12}$$

where $a(t)$ is the scale factor. We decompose the values of inflatons into the 0th order homogeneous parts and the first order perturbations as $\phi_i(t, \vec{x}) = \bar{\phi}_i(t) + \delta\phi_i(t, \vec{x})$. Then, the equations which describe the evolution of the scale factor and $\bar{\phi}_i$ are

$$\ddot{\bar{\phi}}_i + 3H\dot{\bar{\phi}}_i + V_i = 0, \tag{13}$$

$$H^2 = \frac{1}{3}\left(\sum_{i=2,3}\frac{1}{2}\dot{\bar{\phi}}_i^2 + V(\bar{\phi}_2, \bar{\phi}_3)\right), \tag{14}$$

where a dot denotes time derivative, a subscript i of V means a derivative of V by ϕ_i and $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

The first order perturbative equations for Φ and $\delta\phi_i$ are written as

$$\delta\ddot{\phi}_i + 3H\delta\dot{\phi}_i - \frac{1}{a^2}\nabla^2\delta\phi_i + \sum_{j=2,3}V_{ij}\delta\phi_j = 2V_i\Phi - 4\dot{\bar{\phi}}_i\dot{\Phi}, \tag{15}$$

$$\ddot{\Phi} + 5H\dot{\Phi} - \frac{1}{3a^2}\nabla^2\Phi + \frac{4}{3}V\Phi = \frac{1}{3}\sum_{j=2,3}\left(2V_j\delta\phi_j - \dot{\bar{\phi}}_j\delta\dot{\phi}_j\right). \tag{16}$$

8

Hereafter, we consider the Fourier modes of perturbative quantities,

$$\Phi(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left(\Phi(t, \vec{k})e^{i\vec{k}\cdot\vec{x}} + \Phi^\dagger(t, \vec{k})e^{-i\vec{k}\cdot\vec{x}} \right), \tag{17}$$

$$\delta\phi_i(t, \vec{x}) = \int \frac{d^3k}{(2\pi)^3} \left(\delta\phi_i(t, \vec{k})e^{i\vec{k}\cdot\vec{x}} + \delta\phi_i^\dagger(t, \vec{k})e^{-i\vec{k}\cdot\vec{x}} \right). \tag{18}$$

Since they have quantum nature, the Fourier modes are written in terms of creation and annihilation operators as

$$\delta\phi_i(t, \vec{k}) = \sum_{j=2,3} \psi_{ij}(t, \vec{k})a_j(\vec{k}), \qquad \Phi(t, \vec{k}) = \sum_{j=2,3} f_j(t, \vec{k})a_j(\vec{k}), \tag{19}$$

where $a, a^\dagger$ satisfy the following commutation relations:

$$[a_i(\vec{k}), a_j^\dagger(\vec{k}')] = (2\pi)^3 \delta_{ij}\delta^3(\vec{k} - \vec{k}'). \tag{20}$$

When the mass-squared matrix V_{ij} has non-diagonal components, mixing of fields occurs and each $\delta\phi_i(t, \vec{k})$ is written by the linear combination of various annihilation operators. In the case under consideration, the non-diagonal components of V_{ij} are suppressed by one more factor of $\mathcal{V}^{-1}$ compared with diagonal ones. As a result, non-diagonal components of ψ_{ij} are also suppressed unless they are amplified by the resonance effect discussed in the next section. Using this decomposition, (15) and (16) are written as

$$\ddot{\psi}_{ij} + 3H\dot{\psi}_{ij} + \frac{k^2}{a^2}\psi_{ij} + \sum_{k=2,3} V_{ik}\psi_{kj} = 2V_i f_j - 4\dot{\bar{\phi}}_i\dot{f}_j, \tag{21}$$

$$\ddot{f}_i + 5H\dot{f}_i + \frac{k^2}{3a^2}f_i + \frac{4}{3}V f_i = \frac{1}{3}\sum_{j=2,3} \left(2V_j\psi_{ji} - \dot{\bar{\phi}}_j\dot{\psi}_{ji} \right). \tag{22}$$

The initial condition for ψ_{ij} is given as follows,

$$\psi_{ij} \xrightarrow{t\to 0} \frac{1}{\sqrt{2k}a(t)} \exp\left(-ik \int^t \frac{dt'}{a(t')} \right) \delta_{ij}. \tag{23}$$

That is, we assume that in the short wavelength limit $\frac{k}{a} \to \infty$, each scalar field approaches the massless state in the Minkowski space-time. The initial condition for Φ is given through the Poisson equation as

$$\Phi = \frac{a^2}{2k^2} \sum_{j=2,3} \left(\dot{\bar{\phi}}_j\dot{\delta\phi}_j + 3H\dot{\bar{\phi}}_j\delta\phi_j + V_j\delta\phi_j \right) \tag{24}$$

$\mathcal{V}$	W_0	a_2	a_3	A_2	A_3	λ_2	λ_3	α	ξ	g_s
2.45×10^4	1.81	$\frac{2\pi}{30}$	$\frac{2\pi}{5}$	1.13×10^{-3}	0.452	0.3	1.0	1.0	0.5	0.1

Table 1: the parameter set used for the numerical calculation.

Φ should also satisfy the following constraint:

$$\dot{\Phi} = -H\Phi - \frac{1}{2} \sum_{j=2,3} \dot{\phi}_j \delta\phi_j. \tag{25}$$

We use this constraint to check the precision of our numerical calculation.

The perturbations of the metric and inflatons are related to the curvature perturbation as

$$\mathcal{R} = \Phi - \frac{H}{\dot{\sigma}}\delta\sigma, \tag{26}$$

where

$$\dot{\sigma} = \sqrt{\sum_{j=2,3} \dot{\phi}_j^2}, \qquad \delta\sigma = \sum_{j=2,3} \frac{\dot{\phi}_j}{\dot{\sigma}}\delta\phi_j \tag{27}$$

The power spectrum of the curvature perturbation $\mathcal{P}_\mathcal{R}$, which is defined as $\langle |\mathcal{R}|^2 \rangle = \int_0^\infty \frac{dk}{k} \mathcal{P}_\mathcal{R}(k)$, where $\langle \cdots \rangle$ denotes the VEV, is given by

$$\mathcal{P}_\mathcal{R}(k) = \frac{k^3}{2\pi^2} \sum_{j=2,3} \left| f_j - \frac{H}{\dot{\sigma}^2} \sum_{k=2,3} \dot{\phi}_k \psi_{kj} \right|^2 \tag{28}$$

4 Power spectrum of the curvature perturbation

Given the scalar potential in Sec. 2 and the formalism to compute the perturbations in Sec. 3, we now proceed to calculate the power spectrum of the curvature perturbation. In the following, we use the parameter set shown in Table 1. Under this parameter set, the global minimum of the potential is located at

$$\phi_{2,min} = 2.24 \times 10^{-2}, \qquad \phi_{3,min} = 3.06 \times 10^{-2} \tag{29}$$

4.1 Evolution of homegeneous parts

The evolution of the homogeneous parts of inflatons can be traced by numerically integrating (13). We set the initial values of inflatons as

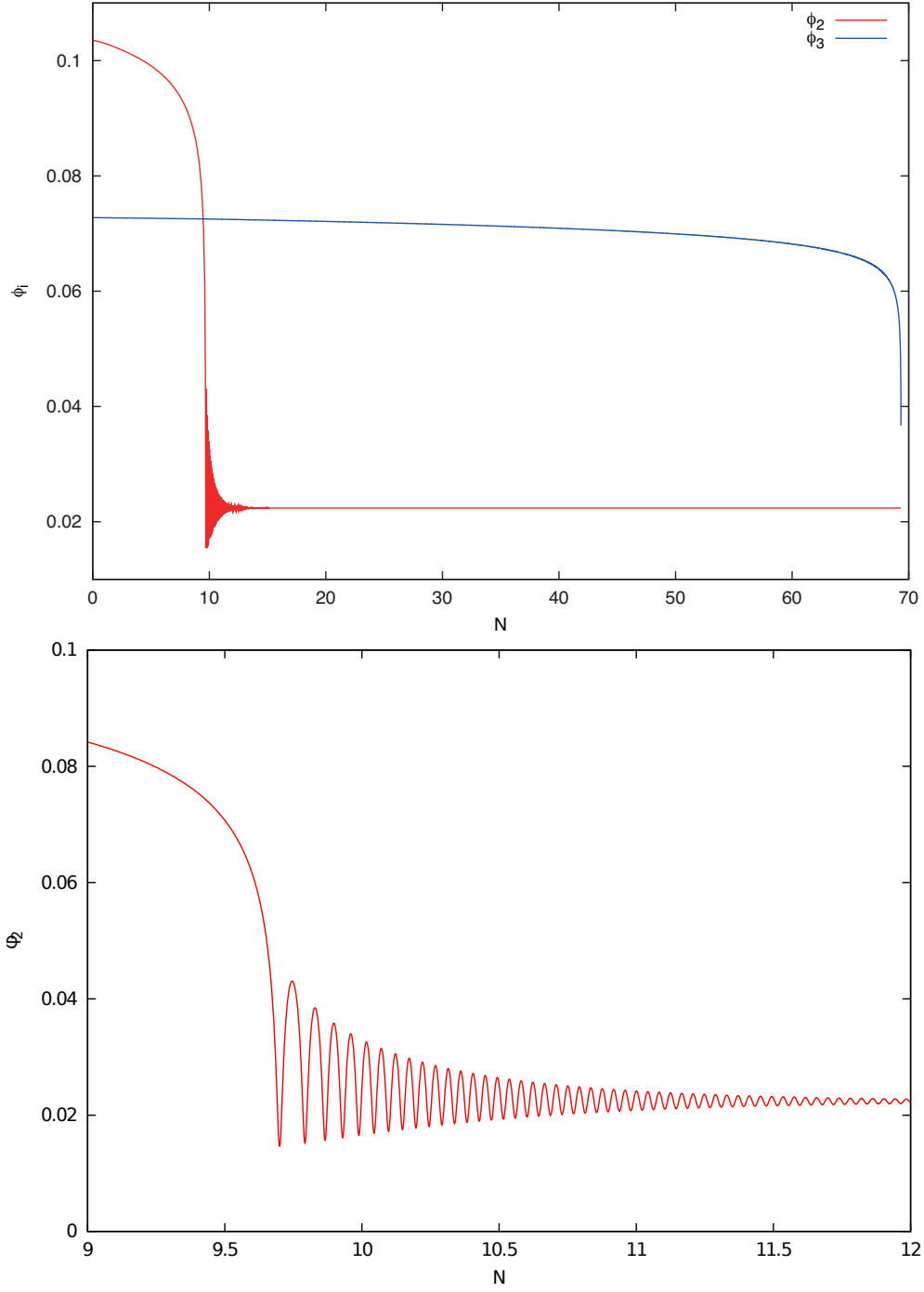$$\tau_{2,ini} = 75.5, \qquad \tau_{3,ini} = 21.15, \tag{30}$$

Figure 2: Evolution of homogeneous parts of inflatons. The red line corresponds to $\bar{\phi}_2$ and blue one to $\bar{\phi}_3$. The upper panel shows the motion of inflatons throughout the inflation and the lower panel focuses on the rapid oscillation of ϕ_2 after it falls down to its minimum.

and those of time derivatives of inflatons to zero. As in most double inflation models, we have to tune the initial position of the second inflaton τ_3 such that the second stage of the inflation is not too long and the signal from the first stage remains in the observable region. The evolution until ϕ_3 falls into its minimum is described in Fig.2. The horizontal axis denotes the e-folding number counted from the start of the evolution. We find that two inflaton actually evolve independently. After the first stage of the inflation $\bar{\phi}_2$ oscillates about its minimum and its amplitude decreases as the universe expands. Then, the potential energy of ϕ_3 dominates the universe and the second inflation takes place.

The energy scale of the inflation in the first stage and the second stage are, $V_1^{1/4} \simeq 1.60 \times 10^{-4}$, $V_2^{1/4} \simeq 1.46 \times 10^{-4}$, respectively. Therefore, no significant gravitational waves cannot be generated in this model.

4.2 Evolution of perturvations

Then, let us see how cosmological perturbations evolve and how the power spectrum is generated by the inflation. They are found by numerically integrating (21) and (22) until the Fourier modes of interest exit the horizon and using the formula (28). The resultant power spectrum of the curvature perturbation is shown in Fig. 3. We assume that the mode whose wavenumber is equal to $k = 0.002 Mpc^{-1}$ at present exits the horizon at the time when the e-folding number counted backward from the end of the inflation is $N = 56.$[4]

In the spectrum one can see the existence of a gentle step at $k \sim 10^{-4}$ Mpc^{-1}. The perturbations with wavenumber larger than 10^{-4} Mpc^{-1} exit the horizon during the first inflation, while thoese with smaller wavenumber exit during the second inflationary stage. Since the Hubble parameter is larger and both inflatons contribute to the curvature perturbation in the first inlationary stage, the amplitude of the perturbation generated during the first inflation is larger, which makes the gentle step at $k \sim 10^{-4}$ Mpc^{-1}.

In addition to the step in the spectrum, there is a small peak at $k \simeq 3 \times 10^{-3}$ Mpc^{-1}. Except these points, the shape is same as the power spectrum generated by the usual slow-roll inflation, which is nearly scale-invariant and can be approximated by power law, $\mathcal{P}_{\mathcal{R}}(k) \propto k^{n_s-1}$. If we neglect the peak, the amplitude and the spectrum index are

$$\mathcal{P}_{\mathcal{R}} \simeq 2.43 \times 10^{-9}, \qquad n_s \simeq 0.965 \tag{31}$$

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[4]We assume that the peak of the power spectrum is located at the scale which is of observational interest as discussed in the next section.
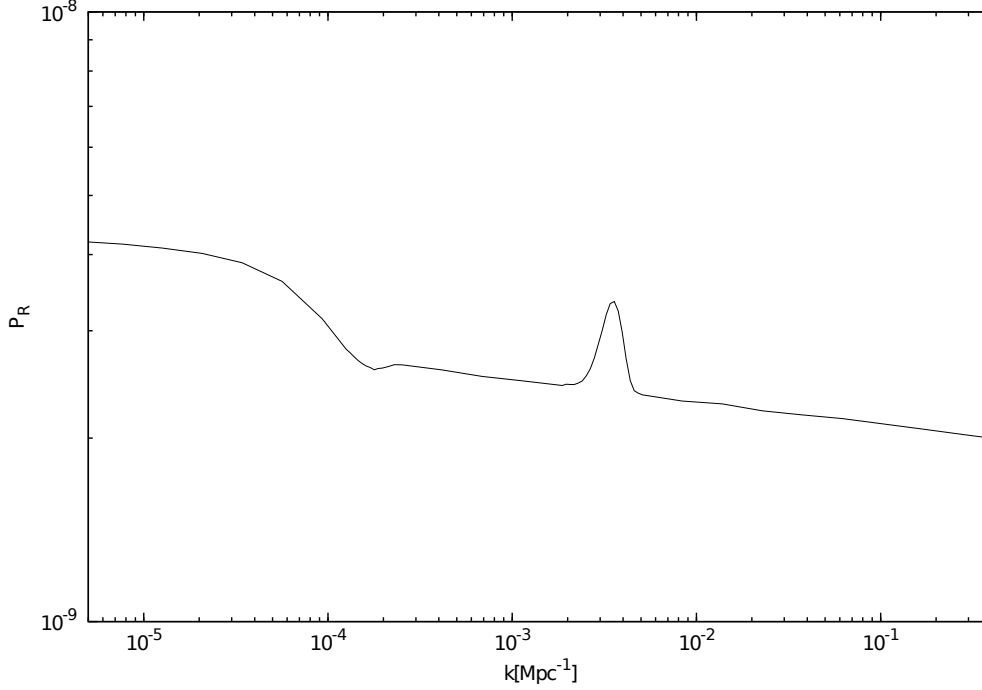
Figure 3: The power spectrum of the curvature perturbation generated by the inflation model in this paper. The horizontal line denotes the wavenumber at present.

at $k = 0.002 Mpc^{-1}$ which are consistent with WMAP 7-year data [33].

Then, let us consider the physics behind the peak in the power spectrum. In order to understand it, we need to see how the fluctuations of the two inflatons evolve. The evolution of $|\psi_{ij}|^2$ and $\langle |\delta\phi_i|^2 \rangle = \sum_j |\psi_{ij}|^2$ are shown in Fig. 4 and Fig. 5, respectively.

Before the onset of the oscillation of ϕ_2, both $\delta\phi_2$ and $\delta\phi_3$ are fluctuations of nearly massless scalar fields, which can be approximated by (23). This is why the evolutions of ψ_{22} and ψ_{33}, or ψ_{23} and ψ_{32} are identical before $N \simeq 10$. However, $\psi_{22}, \psi_{23}, \psi_{32}$ are exponentially enhanced for $10 \lesssim N \lesssim 11$, while ψ_{33} continues decreasing. The period during which they are enhanced depends on how the amplitude of ϕ_2 decreases. Ignoring metric perturbations, the evolution equations of ψ_{ij} are

$$\ddot{\psi}_{22} + 3H\dot{\psi}_{22} + \frac{k^2}{a^2}\psi_{22} + V_{22}(\phi_2, \phi_3)\psi_{22} + V_{23}(\phi_2, \phi_3)\psi_{32} = 0, \tag{32}$$

$$\ddot{\psi}_{23} + 3H\dot{\psi}_{23} + \frac{k^2}{a^2}\psi_{23} + V_{22}(\phi_2, \phi_3)\psi_{23} + V_{23}(\phi_2, \phi_3)\psi_{33} = 0, \tag{33}$$

$$\ddot{\psi}_{32} + 3H\dot{\psi}_{32} + \frac{k^2}{a^2}\psi_{32} + V_{32}(\phi_2, \phi_3)\psi_{22} + V_{33}(\phi_2, \phi_3)\psi_{32} = 0. \tag{34}$$
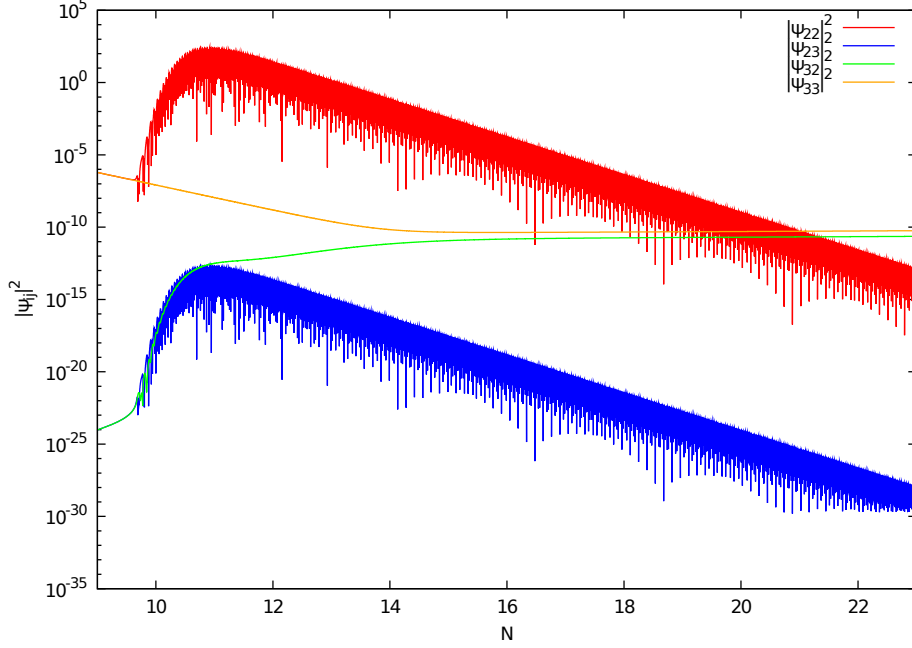
13

Figure 4: The evolution of $|\psi_{ij}|^2$. The red, blue, green, orange lines correspond to $\psi_{22}, \psi_{23}, \psi_{32}, \psi_{33}$ respectively. The horizontal line denotes the e-folding number counted from the beginning of the inflation.

$$\ddot{\psi}_{33} + 3H\dot{\psi}_{33} + \frac{k^2}{a^2}\psi_{33} + V_{32}(\phi_2, \phi_3)\psi_{23} + V_{33}(\phi_2, \phi_3)\psi_{33} = 0. \tag{35}$$

Remembering that the diagonal components of V_{ij} are greater than the non-diagonal ones by a factor $\mathcal{V}$ and so are those of ψ_{ij}, the dominant term in (32) is the fourth term. Since ϕ_2 oscillation makes V_{22} oscillate ψ_{22} has the oscillating mass term. This leads to the exponential growth of ψ_{22} by the parametric resonance. The reason why ψ_{23} grows is similar and the responsible term in (33) is fourth one. The rapid growth of the fluctuation of an inflaton due to parametric resonance in single Kähler moduli inflation was pointed out in [39] where the possibility that the backreaction by the created particle by the resonance immediately becomes important was also pointed out. If the energy stored in the amplified perturbation exceeds or is comparable to that of background condensation $V(\bar{\phi}_2, \bar{\phi}_3)$, we have to take into account the backreaction seriously. However, under the parameter set of Table 1, we find that the energy of the perturbation does not exceeds $V(\bar{\phi}_2, \bar{\phi}_3)$, which allows us to ignore the backreaction. As the amplitude of the oscillation of $\bar{\phi}_2$ decreases, the parameteric resonance becomes inefficient and eventually the growth of ψ_{22} and ψ_{23} stops. After that, their amplitudes decrease by the cosmic expansion. The
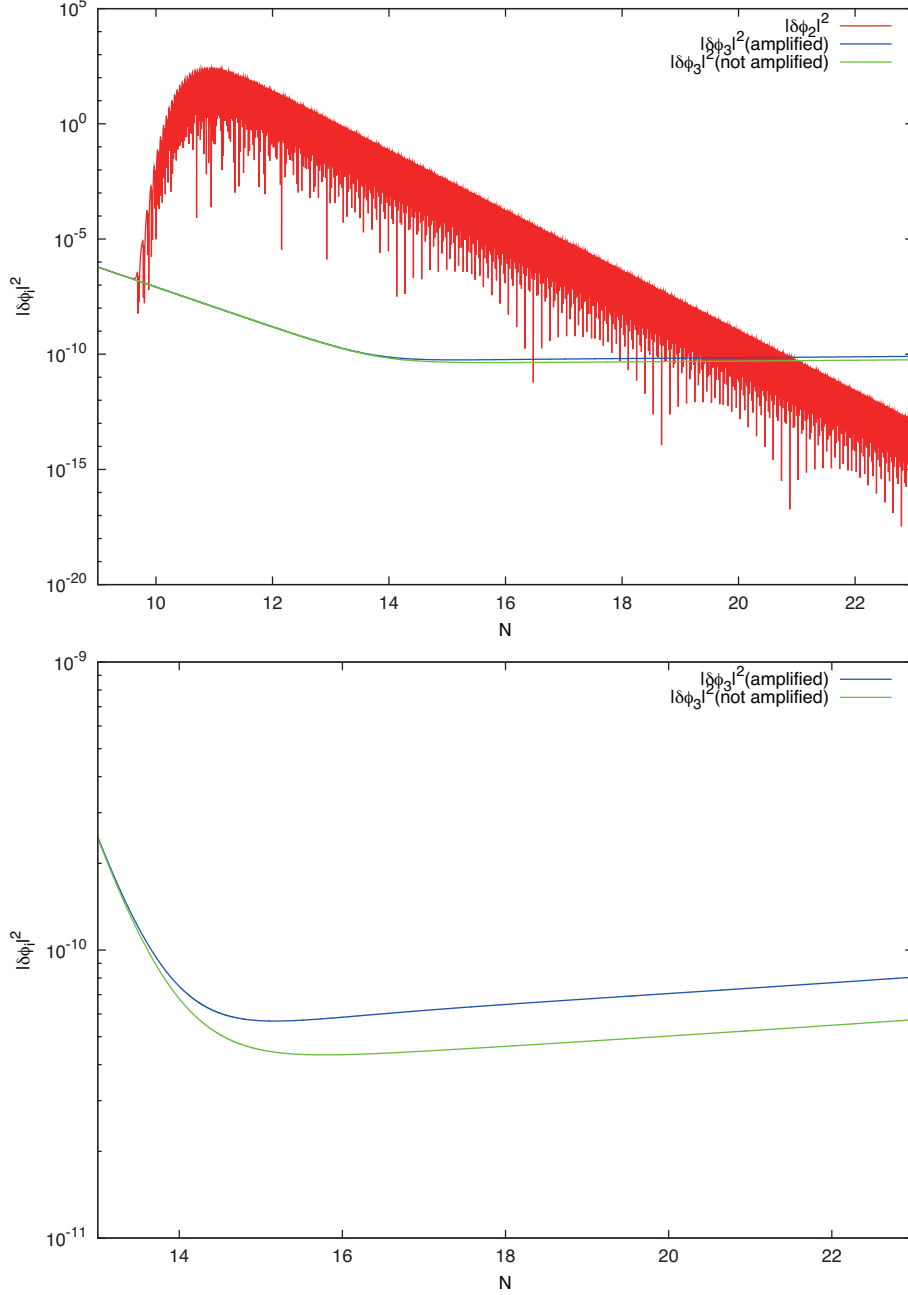
Figure 5: The upper panel shows the evolution of $\langle |\delta\phi_i|^2 \rangle = \sum_j |\psi_{ij}|^2$. The red and blue lines correspond to $\delta\phi_2$ and $\delta\phi_3$ respectively. The green line represents the evolution of $\langle |\delta\phi_3|^2 \rangle$ without the amplification by the resonance. The lower panel focus on the difference between the ways for $\langle |\delta\phi_3|^2 \rangle$ to evolve with and without the amplification. The horizontal line denotes the e-folding number counted from the beginning of the inflation.

total fluctuation of ϕ_2, $\langle|\delta\phi_2|^2\rangle = \sum_j |\psi_{2j}|^2$ shows similar behavior as seen in Fig. 5, that is, it once grows and finally decreases. Therefore, it does not contribute to the final value of the curvature perturbation which exits the horizon in the second stage of the inflation.

ψ_{32} grows in a different way from ψ_{22} and ψ_{23}. Because of the exponential growth of ψ_{22}, (34) is dominated by the fourth term, $V_{32}\psi_{22}$. Then, this term behaves as the source term and ψ_{32} experiences the forced oscillation, which amplifies ψ_{32}. ψ_{32} continues growing even after ψ_{22} stops growing, since the amplitude of ψ_{22} is still large. The situation that the fluctuation of the second inflaton is amplified through forced oscillation by that of the first inflaton enhanced by the resonance effect is similar to that in [29]. Under the present parameters, ψ_{32}, which is originally suppressed compared with ψ_{33}, is amplified as large as ψ_{33}.[5] As a result, the total fluctuation of ϕ_3, $\langle|\delta\phi_3|^2\rangle = \sum_j |\psi_{3j}|^2$ is amplified by a $\mathcal{O}(1)$ factor compared with that in the situation without above effects, as shown in Fig. 5. Since this is the only scalar field fluctuation which survives in the second stage of the inflation, it determins the amplitude of the curvature perturbation which exits the horizon in the second stage. The amplification of the curvature perturbation is also $\mathcal{O}(1)$.

The Fourier modes which experiences amplification are those which exist in the resonance band while $\bar{\phi}_2$ is rapidly oscillating and are somewhat subhorizon. Modes whose wavelength are comparable to or exceed the horizon scale, or much shorter than it are not affected. Therefore the spectrum has a sharp peak as shown in Fig. 3.

In this paper, we choose parameters so that the amplification of perturbation does not become too large in order to neglect the backreaction and discuss the effect of the above power spectrum on the CMB spectrum in the next section.

5 TT-spectrum of CMB based on the power spectrum with small peak

Standard slow-roll inflation models predict a nearly scale-independent power spectrum, which is consistent with various cosmological observations, in particular measurements of CMB anisotropies such as WMAP [33]. However, several data points deviate from the predicted spectrum which is based on a simple power-law power spectrum, $\mathcal{P}_{\mathcal{R}} \propto k^{n_s-1}$. Although such deviations is not statistically significant enough, there is a possibility that

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[5]Since there are neither a largely oscillating mass term nor a source term which induces forced oscillation in the equation of motion of ψ_{33}, it shows same behavior as in the case of the usual slow-roll inflation, that is, it decreases until the mode exits the horizon and after that it remains constant.
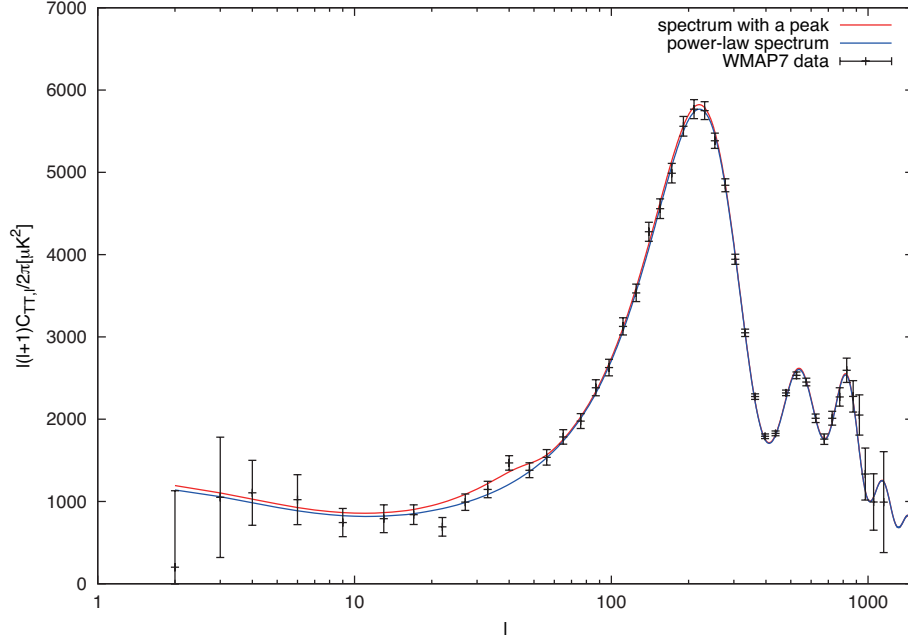
Figure 6: TT-spectra of the CMB. Red one represents the TT-spectrum calculated by CAMB using the power spectrum of Fig. 3. Blue one is the spectrum based on the power-law power spectrum whose amplitude and slope are equal to WMAP-7year best-fit values. Black points are the observed values of WMAP-7year and error bars denote the sum of cosmic variance and systematic errors.

they indicate the feature in the power spectrum which cannot be described by power-law. For example, there is a upward deviation of the observational value around $\ell \simeq 40$. This might suggest that there is a small peak of the power spectrum around the scale corresponding to $\ell \simeq 40$ [42, 43]. In the previous section we have seen that the moduli double inflation predicts a sharp peak in the power spectrum. In fact, we can choose the initial value of the second inflaton so that the peak is located around such scale. Besides, we can tune the $\mathcal{V}$, which determines the strength of the coupling between τ_2 and τ_3, in order to get the desired height of the peak. In Fig. 6 we show the TT power spectrum calculated using the spectrum of Fig. 3 and the computer code CAMB [40]. As for the cosmological parameters other than thoes related power spectrum, we use the WMAP-7year best-fit values [41]. The TT spectrum base on the simple power-law power spectrum and best-fit parameters of WMAP-7year is also shown in Fig. 6.

The resultant TT-spectrum actually has a small bump around $\ell \simeq 40$, which fits the

data points better than that based on the simple power-law power spectrum. We can calculate the likelihood of each TT-spectrum using the code which is available at the website of LAMBDA [44]. We get $-2\ln\mathcal{L} \simeq 1236$ for that based on the power-law power spectrum, and $-2\ln\mathcal{L} \simeq 1233$ for that based on the spectrum of Fig. 3. Thus, there is actually slight improvement of the fit, $\Delta(-2\ln\mathcal{L}) \simeq 3$. Note that in the model under consideration there are two more parameters which can be tuned to improve the fit to WMAP than the simplest slow-roll inflation model, that is, the initial position of τ_3 and $\mathcal{V}$.

6   Summary

In this paper, we have considered Kähler moduli inflation and pointed out that the double inflation takes place when we introduce two Kähler moduli in the inflationary dynamics. Because of plethora of moduli in the general string compactification scenario, the double inflation or the inflation which consists of many stages is natural rather than possible. In the Kähler moduli inflation, the small coupling between the two Kähler moduli is automatically introduced. We have found that through this coupling, the oscillation of the first inflaton in the intermediate period between the two inflationary stages amplifies the fluctuation of the second inflaton and that this results in the peak of the power spectrum of the curvature perturbation. It is interesting that even if all Fourier modes of observational interest exit the horizon in the final stage of the inflation, the information of the previous iflation can be left in the observable power spectrum. We have numerically calculated the power spectrum with a peak under a specific parameter set and applied it to TT-spectrum of the CMB. We have seen that the peak makes the TT-spectrum better fitted to the data of WMAP-7year, which has an upward deviation from the prediction of the simple power-low spectrum around $\ell \simeq 40$.

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